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ABSTRACT

This advanced mathematics curriculum guide, produced under the direction of the State of Louisiana Department of Public Education, is a segment of the educational program established in response to accountability, assessment, and competency-based education laws. The guide is designed to represent the best thinking of a selected statewide committee established to determine the scope of mathematics content for a course following geometry and two years of algebra. The guide contains: (1) the membership rosters of the committees involved in developing the material: (2) a review of the curriculum development process: (3) seventeen required and three optional goals that students completing the advanced mathematics course should be able to reach: (4) a complete curriculum outline with performance objectives: and (5) a detailed set of activities grouped with specific objectives and content areas of the advanced mathematics program. (MP)

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Advanced Mathematics Curriculum Guide

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Louisiana Department Of Education

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ADVANCED MATHEMATICS

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Division of Academic Affairs

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FOREWORD

Curriculum guides have been developed for each mathematics course at the secondary level and for grades K-8 at the elementary level. These guides represent the best thinking of a selected statewide committee established to determine the scope of mathematics content which should be taught at each level.

The mathematics curriculum guides are another segment of the total educational program established by this administration and mandated by the Legislature in both the accountability and assessment and the competency-based education laws. This educational program requires that specific skills and concepts be established for each grade level and for each subject area. The mathematics curriculum guides with course outlines, performance objectives and coordinated activities effect this phase of the program.

It is hoped that the guides will make a major contribution to the improvement of mathematics instruction in the schools of Louisiana. This is another step toward achieving the goals of this administration.

FOR OUR CHILDREN

J. KELLY NIX

ACKNOWLEDGMENTS

The statewide mathematics committee is to be commended for its work in the development of the Mathematics Curriculum Guides K-12. The committee worked under the chairmanship of Dr. Jean Reddy, Section Chief of the Mathematics Section in the Bureau of Secondary Education.

The Bureaus of Elementary Education and Secondary Education were responsible for writing the activities component of the Mathematics Curriculum Guides. The elementary supervisors in the Bureau of Elementary Education with Mrs. Bonnie Ross serving as chairman of the committee, developed the activities for the K-8 guide. The activities for the secondary guides were drafted by a committee under the leadership of Dr. Jean Reddy. These people are to be commended for their colossal accomplishments in this formidable project.

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INTRODUCTION

Act 750 of the 1979 Louisiana Legislature established the Louisiana Competency-Based Education Program. One of the most important provisions of Act 750 is the mandated "development and establishment of statewide curriculum standards for required subjects for the public elementary and secondary schools of this state...." The "statewide curriculum standards for required subjects" is defined as "the required subjects to be taught, curriculum guides which contain mimimum skills and competencies, suggested activities, suggested materials of instruction, and minimum required time allotments for instruction in all subjects." Act 750 further provides that the "effective implementation date of the statewide curriculum standards for required subjects shall be the 1981-82 school year. Development of such curriculum shall begin by the 1979-80 school year."

During the 1978-79 school year, curriculum guides were developed by advisory and writing committees representing all levels of professional education and all geographic areas across the State of Louisiana for the following mathematics courses:

> Algebra I, Algebra II, Geometry, Advanced Mathematics, and Trigonometry.

The major thrust of the curriculum development process in each of the guides has been the establishment of minimum standards for student achievement. Learning expectancies for mastery have been determined for each course and/or grade level. In addition, content outlines, suggested activities, procedures, and bibliographies have been developed as aids in support of the learning expectancies. The curriculum guides also contain activities designed to stimulate learning for those students capable of progressing beyond the minimums.

During the 1979-80 school year, the curriculum guides were piloted by teachers in school systems representing the different geographic areas of the state as well as urban, suburban, inner-city, and rural schools. The standard populations involved in the piloting reflected also the ethnic composition of Louisiana's student population. Participants involved in the piloting studies utilized the curriculum guides to determine the effectiveness of the materials that were developed. Based upon the participants' recommendations at the close of the 1979-80 pilot study, revisions were made in the curriculum guides to ensure that they are usable, appropriate, accurate, comprehensive, relevant, and clear.

The curriculum guides are now ready for full program implementation. This stage must be understood in its operational context. The curriculum developers and the participants in the pilot studies do not stand alone in promoting learning expectancies that will improve education for the students in the State of Louisiana. Ultimately, local system supervisors, principals, and classroom teachers will have the responsibility for attaining this goal.

As curriculum guides are implemented, the following guidelines should prove helpful:

> ...curriculum standards should be considered as the foundation for the year's instructional program. Where other programs are already in operation, these curricular materials must be checked with the foundation curricula to ensure that appropriate course and/or grade level standards are included and maintained.

...curricular activities contained in the guides provide a number of suggestions for helping students to achieve the established standards. Activities to meet the needs of "average," "below average," and "above average" students have been included. These activities should prove helpful as the teacher plans and organizes instruction. Additional activities, however, may supplement or be used in lieu of those listed in the guide as long as these activities are designed to achieve similar specific objectives.

...curricular suggestions for meeting the needs of the special child have been prepared by the Division of Special Education. These suggestions are designed to provide help for teachers who work with special children in the regular classroom.

The continued effort of mathematics teachers to provide quality instruction will enhance our statewide goal to ensure that every student in the public elementary and secondary schools of the State of Louisiana has an opportunity to attain and to maintain skills that are considered essential to functioning effectively in society.

J. KELLY NIX
State Superintendent of Education

GOALS

Upon completing a high school course in advanced mathematics, a student will be able to:

1. Acquire a knowledge of an angle and its measure as applicable to trigonometry.
2. Know basic trigonometry definitions and formulas and be able to apply them.
3. Find the trigonometric functions of any given angle.
4. Graph the six trigonometric functions and their variations.
5. Solve problems involving inverse trigonometric relations.
6. Use identities in solving trigonometric equations.
7. Understand the system of polar coordinates.
8. Know the properties of complex numbers.
9. Solve triangles.
10. Understand the relationship between trigonometric functions and circular functions.
11. Require a knowledge and understanding of circular functions.
12. Recognize a function and specify its domain and range.
13. Recognize the techniques related to polynomial functions.
14. Identify and graph the conic section, and interpret given data to compute determined equations.
15. Distinguish between types of sequences and series and use related techniques and skills.
16. Perceive and apply the process involved in relating exponential and logarithmic functions.
17. Determine and apply appropriate methods for operations with matrices and the evaluation of determinants.

OPTIONAL GOALS

18. Illustrate and use vector concepts and notation.
19. Identify, apply and compute the probability of given events.
20. Find simple derivatives to find the area of given regions.

CURRICULUM OUTLINE AND PERFORMANCE OBJECTIVES

NOTE: All items are mandatory unless preceded by an asterisk. All items with an asterisk should be taught if time permits (See Pacing Chart).

CURRICULUM OUTLINE	PERFORMANCE OBJECTIVE
I. Circular Functions	To display an understanding of circular functions, the student will be able to:
A. Unit circle	A. Define the unit circle and demonstrate symmetry with respect to the two axes and the origin.
B. Wrapping function	B. Define and illustrate the wrapping function.
C. Sine and cosine functions	C. Define sine and cosine functions as coordinates of a point on the unit circle and state the domain, range and fundamental period.
D. Graphs of functions of the form $y = \sin x$ and $y = \cos x$	D. Graph two or more periods of functions of the form $y = \sin x$ and $y = \cos x$.
E. Graphs of functions of the form $y = a \sin bx$ and $y = a \cos bx$	E. Graph functions of the form $y = a \sin bx$ and $y = a \cos bx$ by finding the period and/or amplitude of each.
F. Graphs of functions of the form $y = a \sin (bx - c) + d$ and $y = a \cos (bx - c) + d$	F. Graph functions of the form $y = a \sin (bx - c) + d$ and $y = a \cos (bx - c) + d$ by finding the period and/or amplitude of each.
G. Other circular functions	G. 1. Define tangent, cotangent, secant and cosecant in terms of the sine and cosine; 2. Graph and state the domain and range of each.
H. Identities	H. State the eight basic identities and use them to verify other identities.

CURRICULUM OUTLINE	PERFORMANCE OBJECTIVES
II. Angles And Their Measures	To demonstrate a knowledge of basic definitions, the student will be able to:
A. Definition of an angle	A. Define an angle and identify the initial and terminal sides of an angle.
B. Standard positions of an angle	B. Sketch angles in standard position.

CURRICULUM OUTLINE	PERFORMANCE OBJECTIVES
III. Radian Measure	To demonstrate an understanding of degree measure and radian measure, the student will be able to:
A. Conversions	A. Convert radian measure to degree measure and degree measure to radian measure.
B. Length of an arc of a circle	B. Use radian measure to find the length of an arc of a circle;
C. Area of sector	C. Use radian measure to find the area of a sector of a circle;
D. Velocity	D.
	1. Compute linear speed of a point moving on a circle;
	2. Find angular velocity of a point moving on a circle.

CURRICULUM OUTLINE	PERFORMANCE OBJECTIVES
IV. Trigonometric Functions	To exhibit a knowledge of a trigonometric functions, the student will be able to:
A. Functions of angles	A. Write the six trigonometric functions of any angle.
B. Functions of special angles	B. Write the trigonometric functions of angles whose measures are 30, 45, and 60 degrees.
C. Cofunctions	C. Write functions of acute angles as functions of the complement of an angle.
D. Reciprocal relations	D. Write functions of angles as reciprocal functions of the same angle.
E. Quotient and Pythagorean relations	E. Write trigonometric expressions as equivalent expressions by using the quotient and Pythagorean relations.
F. Tables and linear interpolation	F. 1. Find functions of angles by reading from a table; 2. Find functions of angles by linear interpolation.

CURRICULUM OUTLINE	PERFORMANCE OBJECTIVES
V. Trigonometric Functions of Any Angle	To demonstrate a knowledge of special and quadrantal angles, the student will be able to:
A. Reference angles	A. Express the functions of any angle as a function of a reference angle.
B. Quadrantal angles	B. Find the functions of angles whose measures are multiples of 0, 90, 180, and 270 degrees.
C. Formulas	C.
1. Sum and difference formulas	1. Apply the formulas for the functions of the sum or difference of two angles;
2. Half-angle and double-angle formulas	2. Apply the double-angle and half-angle formulas.
D. Identities	D. Verify that expressions are identities by using trigonometric formulas.

CURRICULUM OUTLINE	PERFORMANCE OBJECTIVES
VI. Inverse Trigonometric Relations	To display an understanding of the inverse trigonometric relations, the student will be able to:
A. General value	A. Find general values of inverse relations.
B. Graph of inverse	B. Graph inverse trigonometric functions.
C. Principal values of inverse functions	C. Find principal values of inverse trigonometric functions.

CURRICULUM OUTLINE	PERFORMANCE OBJECTIVES
VII. Trigonometric Equations	To demonstrate an understanding of trigonometric identities and equations, the student will be able to:
A. Principal value	A. Find principal values of trigonometric equations.
B. General values	B. Find general values of trigonometric equations.
C. Restricted solutions	C. Find restricted solutions of trigonometric equations.

CURRICULUM OUTLINE	PERFORMANCE OBJECTIVES
VIII. Applications of Trigonometry	To exhibit an understanding of the use of trigonometric ratios in finding missing parts of triangles, the student will be able to:
A. Right triangle	A. 1. Solve right triangles given two sides and an angle; 2. Solve right triangles given two angles and a side.
B. Law of sines and law of cosines	B. Solve triangles by applying the law of sines and the law of cosines.
C. Use of calculators	C. Perform calculations involving trigonometric functions using the calculator.
D. Area of triangles	D. Find the area of a triangle given the lengths of three sides or the lengths of two sides and the measure of the included angle of the triangle.

CURRICULUM OUTLINE	PERFORMANCE OUTLINE
IX. Polar Coordinates	To display an understanding of the polar coordinate system, the student will be able to:
A. Conversion	A. Convert from rectangular to polar coordinates and vice-versa.
B. Graphing	B. Sketch graphs of relations in polar form.

CURRICULUM OUTLINE		PERFORMANCE OBJECTIVES
X.	Complex Numbers	To exhibit a knowledge of complex numbers, the student will be able to:
	A. Definition and properties	A.
		1. Define a complex number;
		2. Perform fundamental operations on complex numbers.
	B. Rectangular and polar forms	B. Write complex numbers in rectangular and polar form.
	C. Multiplication and division in polar form	C. Multiply and divide complex numbers in polar form.
	D. DeMoivre's theorem	D. Use DeMoivre's Theorem to raise complex numbers to integral powers.
	E. Roots of complex numbers	E. Find all roots of a complex number.

NOTE: All items are mandatory unless preceded by an asterisk. All items with an asterisk should be taught if time permits (See Pacing Chart).

CURRICULUM OUTLINE	PERFORMANCE OBJECTIVES
XI. Relations and Functions	To demonstrate an understanding of relations and functions, the student will be able to:
A. Relations	A. Define a relation and identify the domain and range.
B. Linear relations and graphs	B. Define a linear relation and sketch its graph.
C. Functions	C. 1. Identify a function from a graph, set of ordered pairs, equation, or mapping; 2. Sketch and state the domain and range of special functions.
D. Arithmetic of functions	D. Perform arithmetic operations with functions and their graphs.
E. Composition and inversion of functions	E. Find a composition of two functions and express the inverse of a given function.
F. Exponential and logarithmic functions	F. 1. Change from exponential to logarithmic form and conversely; 2. Sketch these functions.

CURRICULUM OUTLINE	PERFORMANCE OBJECTIVE
XII. Polynomial Functions	To demonstrate an understanding of polynomial functions, the student will be able to:
A. Definition	A. Define a polynomial in x.
B. Factor and remainder theorems	B. Apply the Factor and Remainder Theorems.
C. Synthetic division	C. Use synthetic division to find quotients and remainders.
D. Rational root theorem	D. Use the Rational Root Theorem to find roots of equations.
E. Location theorem	E. Locate real roots of equations that are between two consecutive integers.
F. Graphs	F. Graph higher degree equations.
G. Descartes rules of Signs	G. Apply Descartes Rules of Signs to determine the nature of roots of equations.
H. Upper and lower bound theorems	H. Use the Upper and Lower Bound Theorems to solve equations.
I. Writing equations	I. Write an equation of minimum degree that has selected solutions.

CURRICULUM OUTLINE	PERFORMANCE OBJECTIVE
XIII. Conic Sections	To demonstrate an understanding of conic sections, the student will be able to:
A. Circles B. Parabolas C. Ellipses D. Hyperbolas	A, B, C, and D. 1. Identify, write the equation in standard form, and sketch the conic sections from the locus definitions; 2. Identify a conic section represented by $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

CURRICULUM OUTLINE		PERFORMANCE OBJECTIVES
XIV.	Sequences and Series	To demonstrate an understanding of sequences and series, the student will be able to:
	A. Definitions	A. Define a sequence and a series.
	B. Notation	B. Use mathematical notation to write sequences and series.
	C. Recursion formulas	C. Use recursion formulas to write sequences.
	D. Arithmetic sequences and series	D. Find missing terms in arithmetic sequences and series.
	E. Geometric sequences and series	E. Find missing terms in geometric sequences and series.
	F. Infinite geometric series	F. Find the sum of infinite geometric series whose common ratio r has the property that $\|r\| < 1$.
	G. Limit of a sequence	G. Find the limit of an infinite sequence.
	H. Convergent and divergent series	H. Determine if selected infinite series converge or diverge.
	I. Mathematical induction	I. Use the principle of mathematical induction to solve selected exercises.
	J. Binomial theorem	J. Use the Binomial Theorem to expand binomials.

CURRICULUM OUTLINE	PERFORMANCE OBJECTIVES
XV. Matrices	To demonstrate an understanding of matrices, the student will be able to:
A. Definition	A. Define a matrix.
B. Sums and products	B. Find sums and products of matrix arrays.
C. Transpose	C. Find the transpose of a matrix.
D. Determinant	D. Find the determinant of a matrix by using reduction of minors and cofactors.
E. Inverse	E. 1. Find the inverse of a matrix; 2. Solve linear matrices.
F. Systems of equations	F. Solve linear systems of n equations in n variables.

CURRICULUM OUTLINE	PERFORMANCE OBJECTIVES
XVI. Probability	To demonstrate an understanding of probability, the student will be able to:
A. Permutations	A. Define a permutation and solve problems that are associated with permutations.
B. Combination	B. Solve selected exercises that are associated with combinations.
C. Sample space	C. Define a sample space and complementary events.
D. Binomial Theorem	D. Use the Binomial Theorem to solve selected exercises.
E. Dependent and independent	E. Solve selected exercises that involve independent and dependent events.

CURRICULUM OUTLINE	PERFORMANCE OBJECTIVES
XVII. Vectors	To demonstrate an understanding of vectors, the student will be able to:
A. Definition	A. Define Cartesian products and construct Cartesian lattices for finite sets of integers.
B. Rule or roster	B. Specify a set given a rule or roster.
C. Displacements	C. Depict real numbers as displacements along a real number line.
D. Vectors in a plane	D. 1. Recognize vectors in a plane as ordered pairs of real numbers, and as directed line segments; 2. Add vectors; 3. Construct vector diagrams; 4. Diagram vector sums.
E. Norm of vector	E. Find the norm of a vector.
F. Vector by scalar	F. Multiply a vector by a scalar and illustrate the result by a vector diagram.
G. Inner product	G. 1. Find the inner or dot product of two vectors; 2. Specify the properties of inner products.
H. Parallel and perpendicular components	H. 1. Resolve a vector into components that are parallel or perpendicular to a given vector; 2. Determine the unit vector in a specified direction; 3. Represent physical quantities such as force and velocity by vector diagrams.

CURRICULUM OUTLINE	PERFORMANCE OBJECTIVE
XVIII. Differential Calculus	To demonstrate an understanding of differential calculus, the student will be able to:
A. Limit of a function	A. Find limits of functions, if they exist.
B. Continuous function	B. Determine if a function is continuous at a point.
C. Limit theorems	C. Use the limit theorems to find limits of functions.
D. Derivative of a function	D. Find the derivative of a function by:
1. Definition	1. Using the definition of the derivative of a function;
2. Formulas	2. Applying derivative formulas.
E. Local maxima and minima	E. Find local maxima and local minima.
F. Points of inflection	F. Find points of inflection for which $f'(x) = 0$.
G. Graphs	G. Sketch functions by identifying local maxima and/or local minima.
H. Applications	H. Use the derivative of a function to solve verbal problems.

ACTIVITIES

I. A. CONTENT: Circular Functions; Unit Circle

OBJECTIVE: The student will be able to:

(a) Define a unit circle;

(b) Demonstrate symmetry with respect to the axes and the origin.

ACTIVITIES:

(a) Define a unit circle.

(b) Given a point (a,b) on a unit circle, determine the coordinates of a point on the unit circle which demonstrates symmetry with respect to:

(1) The horizontal axis.

(2) The vertical axis.

(3) The origin.

I. B. CONTENT: Circular Functions; Wrapping Functions

OBJECTIVE: The student will be able to:

(a) Define the wrapping functions;

(b) Specify the domain and range of the wrapping function.

ACTIVITIES:

(a) Define a wrapping function.

(b) Given w(x) to be the wrapping function find the following and state the domain and range of each.

(1) $w(\frac{\pi}{2})$

(2) $w(\pi)$

(3) $w(3\pi)$

(4) $w(-\frac{\pi}{2})$

(5) $w(\frac{5\pi}{2})$

(c) In which quadrant does w(x) lie, where x equals:

(1) -3

(2) $+3$

(3) $\frac{7\pi}{5}$

(4) $\frac{3}{2}$

(5) $\frac{10\pi}{3}$

(6) -25

I. C. CONTENT: Circular Functions; Sine and Cosine Functions

OBJECTIVE: The student will be able to:

(a) Define the sine functions as a set of all ordered pairs (x,v) where v = sin x, and the cosine functions as a set of all ordered pairs (x,u) where u = cos x;

(b) State the domain and range;

(c) State the fundamental period;

(d) Find the values of the sine and cosine of integral multiples of $\frac{\pi}{6}$, and $\frac{\pi}{4}$ and $\frac{\pi}{3}$.

ACTIVITIES:

(a) Sin x = $\frac{4}{5}$ and x lies in the second quadrant, find cos x.

(b) State the domain and range of the sine and cosine function.

(c) Name the numbers x, such that sin x = 1.

(d) Find the values of the following:

(1) $\cos \frac{5\pi}{3}$

(2) $\sin \frac{5\pi}{4}$

(3) $\sin \frac{5\pi}{6}$

(4) $\cos \frac{17\pi}{3}$

(5) $\sin \frac{-23\pi}{4}$

I. D. CONTENT: Circular Functions; Graphs of Functions of the form $y = \sin x$ and $y = \cos x$

OBJECTIVE: The student will be able to sketch two or more periods of $y = \sin x$ and $y = \cos x$.

ACTIVITIES: Sketch the graphs of each of the following over the indicated interval.

(a) $y = \sin x$ from -2π to 2π.

(b) $y = \cos x + \sin x$ from 0 to π.

(c) $y = \cos x$ from $\frac{-\pi}{2}$ to $\frac{5\pi}{2}$.

(d) $y = |\cos x|$ from -2π to 2π.

I. E. CONTENT: Circular Functions; Graphs of Functions of the form $y = a \sin bx$ and $y = a \cos bx$

OBJECTIVE: The student will be able to graph functions of the form $y = a \sin bx$ and $y = a \cos bx$ by finding the amplitude and/or period of each.

ACTIVITIES:

(a) Find the amplitude and period of each of the following:

(1) $y = 2 \sin 3x$.

(2) $y = 4 \cos x$.

(3) $y = -3 \cos \frac{1}{3}x$.

(4) $y = \frac{1}{4} \cos \frac{1}{4}x$.

(b) Sketch two periods of the graphs of:

(1) $y = 2 \sin 3x$.

(2) $y = 3 \cos \frac{1}{2}x$.

(3) $y = 3 \sin \pi x$.

(4) $y = 3 \sin \frac{1}{3}x$.

I. F.

CONTENT: Circular Functions; Graphs of Functions of the form $y = a \sin (bx - c) + d$ and $y = a \cos (bx - c) + d$

OBJECTIVE: The student will be able to graph functions of the form $y = a \sin (bx - c) + d$ and $y = a \cos (bx - c) + d$ by finding the amplitude and/or period of each.

ACTIVITIES: Sketch two periods of the graphs of:

(a) $y = \sin (x - \pi)$.

(b) $y = \cos (2x - \pi)$.

(c) $y = -3 \sin (2x - \frac{\pi}{2}) + 4$.

(d) $y = \cos (3x + \pi)$.

(e) $y = 3 \sin (\frac{1}{2}x - \pi) - 2$.

(f) $y = -2 \cos \frac{1}{2} x$.

I. G.

CONTENT: Circular Functions; Other Circular Functions

OBJECTIVE: The student will be able to:

(a) Define tangent, cotangent, secant, and cosecant in terms of the sine and cosine;

(b) Graph and state the domain and range of each.

ACTIVITIES:

(a) Use the values of cosine and sine to compute a table of values for tangent, cotangent, secant, cosecant of the special numbers between 0 and 2π.

(b) Graph two basic periods:

(1) $2 \tan \frac{x}{2}$.

(2) $y = -2 \cot 3x$.

(3) $y = \tan (\frac{1}{3}x + \frac{\pi}{6})$.

(4) $y = 2 \sec (2x - \frac{\pi}{6}) + 2$.

(5) $2 \csc \frac{1}{2}x$.

I. H.

CONTENT: Circular Functions; Identities

OBJECTIVE: The student will be able to state the basic identities and use them to verify other identities.

ACTIVITIES:

(a) State the eight basic identities.

(1) $\sin x \cdot \csc x = 1$

(2) $\cos x \cdot \sec x = 1$

(3) $\tan x \cdot \cot x = 1$

(4) $\tan x = \frac{\sin x}{\cos x}$

(5) $\cot x = \frac{\cos c}{\sin x}$

(6) $\sin^2 x + \cos^2 x = 1$

(7) $1 + \tan^2 x = \sec^2 x$

(8) $1 + \cot^2 x = \csc^2 x$

(b) Verify.

(1) $\cot x \cdot \sec x = \csc x$

(2) $\sin x (\sec x - \csc x) = \tan x - 1$

(3) $\frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = 2 \sec x$

(4) $\sec x - \cos x = \sin x \cdot \tan x$

II. A.

CONTENT: Angles and Their Measures; Definition of an Angle

OBJECTIVE: The student will be able to identify or define:

(a) An angle;

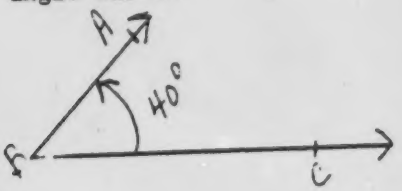
(b) Initial and terminal side of an angle;

(c) Co-terminal angles.

ACTIVITIES:

(a) Define an angle.

(b) Name the initial and terminal sides of the angle shown.



(c) Identify two co-terminal angles for the angle shown in part (b).

II. B.

CONTENT: Angles and Their Measures; Standard Position of an Angle

OBJECTIVE: The student will be able to sketch angles in standard position.

(a) an angle in standard position contains (P(1,1).

Find r (i.e., $\sqrt{x^2 + y^2}$) and the degree measure of the angle.

(b) An angle in standard position contains P(-1,1). Find r and the degree measure of the angle.

(c) An angle in standard position contains P(0,-1). Find r and the degree measure of the angle.

III. A.

CONTENT: Radian Measure; Conversions

OBJECTIVE: The student will be able to convert radian measure to degree measure and degree measure to radian measure.

ACTIVITIES:

(a) Express each of the following in radian measure.

(1) 60°

(2) 135°

(3) 270°

(4) 75°

(5) 300°

(b) Express each of the following radian measures as degree measures.

(1) $\frac{\pi}{2}$

(2) $\frac{5\pi}{4}$

(3) $\frac{11\pi}{6}$

(4) $\frac{7\pi}{8}$

(c) Find numerical values for each of the following without using a table.

(1) $\sin 45^\circ$

(2) $\sin 30^\circ \tan 45^\circ$

(3) $\cos 60^\circ \sec 45^\circ$

(4) $\cot 60^\circ \tan 30^\circ$

(5) $\sin 135^\circ$

III. B. CONTENT: Radian Measure; Length of an Arc of a Circle

OBJECTIVE: The student will be able to use radian measure to find the length of an arc of a circle.

ACTIVITIES:

(a) An arc of a circle has a corresponding central angle whose measure is 120 degrees. Find the length of the arc given that the radius of the circle is 12.

(b) The measure of a central angle of a circle is $\frac{\pi}{6}$ radians.

If the radius of the circle is 8", find the length of the arc.

III. C. CONTENT: Radian Measure; Area of a Sector

OBJECTIVE: The student will be able to use radian measure to find the area of a sector of a circle.

ACTIVITIES:

(a) The radian measure of an arc of a circle is .87. If the radius of the circle is 6 cm, find the area of the sector.

(b) The area of a sector of a circle is 20.3 sq. in. Find the radius of the circle given that the measure of the arc of the sector is $\frac{1}{2}$ radian.

III. D. CONTENT: Radian Measure; Velocity

OBJECTIVE: The student will be able to:

(a) Compute linear speed of a point moving on a circle;

(b) Find angular velocity of a point moving on a circle.

ACTIVITIES:

(a) The blade of a rotary lawn mower is 20 inches in diameter. If the blade turns 360 r.p.m., find, in ft./sec., the linear speed of the tip of the blade.

(b) The wheels of an automobile are 28 inches in diameter. If the automobile travels 65 m.p.h., find the angular velocity of the wheels in radians per second.

IV. A.

CONTENT: Trigonometric Functions; Functions of Angles

OBJECTIVE: The student will be able to write the six trigonometric functions of angles.

ACTIVITIES:

(a) The point P(3,4) lies on the terminal side of the graphed angle D. Write the six trigonometric functions of D.

(b) The point Q(5,-12) lies on the terminal side of the graphed angle B. Write the six trigonometric functions of B.

IV. B.

CONTENT: Trigonometric Functions; Functions of Special Acute Angles

OBJECTIVES: The student will be able to write the six trigonometric functions of angles whose measures are 30°, 45°, and 60°.

ACTIVITIES:

(a) Find numerical values for the following:

(1) $\sin 60^\circ + 2 \cos 30^\circ$

(2) $\sin 30^\circ \cos 60^\circ - \sin 45^\circ \cos 45^\circ$

(3) $2 \sin 60^\circ + \cos 30^\circ - \tan 45^\circ \cot 45^\circ$

(4) $\sec 45^\circ - \csc 30^\circ$

(b) Given $\{180^\circ \leq A < 270\}$ find:

(1) sec A given that $\cos A = \frac{-2}{3}$

(2) tan A given that $\csc A = \frac{-5}{3}$

(3) sin A given that $\cos A = \frac{-3}{8}$

IV. C.

CONTENT: Trigonometric Functions; Cofunctions

OBJECTIVE: The student will be able to write functions of acute angles as functions of the complement of an angle.

ACTIVITIES:

(a) Write each of the following as a function of the complement of the angle:

(1) $\sin 10^\circ = \cos$ _____.

(2) $\tan 80^\circ =$ _____.

(3) cot 25° = _____.

(4) cos 1° + _____.

(5) sec 46° = _____.

(b) Find angle A, given that:

(1) cos A = sin $(45^\circ - \frac{1}{2}A)$

(2) tan $(45^\circ + A)$ = cot A

(3) cos 4A = sin A

IV. D. CONTENT: Trigonometric Functions; Reciprocal Relations

OBJECTIVE: The student will be able to write functions of angles as reciprocal functions of the same angle.

ACTIVITIES:

Use reciprocal relations to write each of the following as a function of the same angle.

(a) sin 10°

(b) tan 40°

(c) sec 15°

(d) csc 70°

(e) cos 25°

(f) cot 80°

IV. E. CONTENT: Trigonometric Functions; Quotient and Pythagorean Relations

OBJECTIVE: The student will be able to write trigonometric expressions as equivalent expressions by using the quotient and pythagorean relations.

ACTIVITIES:

Quotient Relations $\tan B = \frac{\sin B}{\cos B}$

$\cot B = \frac{\cos B}{\sin B}$

Pythagorean Relations $\sin^2 B + \cos^2 B = 1$

$\sec^2 B = 1 + \tan^2 B$

$\csc^2 B = 1 + \cot^2 B$

Use the fundamental identities of this section to answer the following. Assume that each angle is acute.

(a) If $\cos A = \frac{3}{5}$, find sin A. Find tan A.

(b) If $\tan B = \frac{12}{5}$, find cot B.

(c) If $\csc B = \frac{13}{5}$, find cot B.

(d) Express each of the six trigonometric functions as a function of sin A.

IV. F. CONTENT: Trigonometric Functions; Tables and Linear Interpolation

OBJECTIVE: The student will be able to find functions of angles by using a table and by linear interpolation.

ACTIVITIES:

(a) Use a table to find:

(1) sin 20° 10'

(2) tan 84° 40'

(3) cot 48° 20'

(4) cot 14° 50'

(b) Use interpolation to find:

(1) sin 10° 13'

(2) cot 48° 37'

(3) tan 27° 27'

(4) cot 78° 54'

(c) Find acute angle A, given that:

(1) sin A = .0873

(2) cot a = 1.7560

(3) cot A = .5534

(4) tan A = 1.1650

V. A.

CONTENT: Trigonometric Functions of Any Angle; Reference Angle

OBJECTIVE: The student will be able to express the function of any angle as a function of a reference angle.

ACTIVITIES:

(a) Express each of the following as the same function of a first quadrant angle.

(1) $\cot 135^\circ =$

(2) $\tan 220^\circ =$

(3) $\csc \frac{7\pi}{9}$

(4) $\cot 320^\circ =$

(5) $\sin \frac{23\pi}{18}$

(6) $\tan 335^\circ =$

(7) $\sin (-100^\circ) =$

(8) $\tan (-120^\circ) =$

(9) $\sin (-330^\circ) =$

(b) Express each of the following as a function of an angle between 0 and 45°:

(1) $\sin 100^\circ$

(2) $\cot 310^\circ$

(3) $\tan 120^\circ$

(4) $\sec 250^\circ$

(5) $\cot 95^\circ$

(6) $\tan (-100^\circ)$

(7) $\sec (-130^\circ)$

(8) $\tan (-420^\circ)$

(9) $\csc 260^\circ$

(10) $\sin (-225^\circ)$

(c) Find (in simplest form) numerical values for each of the following. Do not use a table.

(1) $\sin 150^\circ \cos 300^\circ$

(2) $\sin 510^\circ \tan 225^\circ$

(3) $\cos (-60^\circ) \csc 315^\circ$

(4) $\sin (-45^\circ) \sin 30^\circ$

(5) $\tan 120^\circ + \cos 120^\circ$

(d) If $0 \leq A < 360^\circ$, find all angles for which:

(1) $\sin a = \frac{1}{2}$

(2) $\tan A = \sqrt{3}$

(3) $\sec A = \frac{2}{\sqrt{3}}$

(4) $\cos A = \frac{\sqrt{3}}{2}$

(5) $\tan A = \sqrt{3}$

(6) $\cos A = -\sqrt{3}$

(7) $\csc A = -\sqrt{2}$

(8) $\tan A = -1$

(9) $\sec A = -\sqrt{2}$

(e)

(1) Find A given that $\sin A = \frac{1}{2}$ and $90^\circ < A < 180^\circ$

(2) Find A given that $\tan A = 3$ and $0^\circ < A < 90^\circ$

(3) Find A given that $\cot A = -3$ and $90^\circ < A < 180^\circ$

(4) Find $\sin A$ given that $\sec A = \frac{5}{4}$ and $270^\circ < A < 360^\circ$

(5) Find $\cos A$ given that $\tan A = \frac{-5}{6}$ and $270^\circ < A < 360^\circ$

(6) Find $\tan A$ given that $\csc A = \frac{-7}{4}$ and $180^\circ < A < 270^\circ$

V. B. CONTENT: Trigonometric Functions of Any Angle; Quadrantal Angles

OBJECTIVE: The student will be able to find the function of angles whose measures are multiples of 0°, 90°, 180°, 270°.

ACTIVITIES: Find numerical values for each of the following:

(a) $\sin 180^\circ$

(b) $\tan 180^\circ \sin 270^\circ + \tan 45^\circ \cos 180^\circ$

(c) $\sec 180^\circ \csc 270^\circ - \cos 270^\circ \sin 90^\circ$

(d) $\cos 540^\circ + \sin 720^\circ$

(e) $\sec 360^\circ \tan (-135^\circ) + \cos 360^\circ \cot (-225^\circ)$

(f) $\sin 180^\circ \cos 45^\circ - \tan 135^\circ \sin 210^\circ + \sin {}^2 90^\circ$

(g) $\sec 60^\circ \cos 90^\circ - \dfrac{\tan 45^\circ}{\cot 225^\circ}$

(h) $\csc (-135^\circ) \sec 180^\circ + \sin 180^\circ \cos 225^\circ$

V. C. CONTENT: Trigonometric Functions of Any Angle; Formulas; Sum and Difference Formulas

OBJECTIVE: The student will be able to apply the formulas for the functions of the sum or difference of two angles.

ACTIVITIES: Formulas:

$\sin (a + B) = \sin a \cos B + \cos a \sin B$

$\sin (a - B) = \sin a \cos B - \cos a \sin B$

$\cos (a + B) = \cos a \cos B - \sin a \sin B$

$\cos (a - B) = \cos a \cos B + \sin a \sin B$

$\tan (a + B) = \dfrac{\tan a + \tan B}{1 - \tan a \tan B}$

$\tan (a - B) = \dfrac{\tan a - \tan B}{1 + \tan a \tan B}$

(a) Find $\sin 105^\circ$ by using $\sin (60^\circ + 45^\circ)$.

(b) Find $\cos 15^\circ$ by using $\cos (45^\circ - 30^\circ)$.

(c) Find $\sin 15^\circ$ by using functions of 45° and 30°.

(d) Find $\sin 75^\circ$ without using a table.

(e) If $\cos A = \frac{4}{5}$, $270^\circ < A < 360^\circ$, and $\sin B = \frac{-12}{13}$, $180^\circ < B < 270^\circ$; find $\cos (A + B)$, $\cos (A - B)$, $\sin (A + B)$, $\sin (A - B)$

(f) If $\cot A = \frac{1}{2}$, $180^\circ < A < 270^\circ$ and $\sin B = \frac{2}{\sqrt{13}}$, $90 < B < 180^\circ$; find $\tan (A + B)$ and $\tan (A - B)$.

V. C. (2) CONTENT: Trigonometric Functions of Any Angle; Formulas; Half-Angle and Double-Angle Formulas

OBJECTIVE: The student will be able to apply half-angle and double-angle formulas.

ACTIVITIES:

Formulas:

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin \left(\frac{A}{2}\right) = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \left(\frac{A}{B}\right) = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}} = \frac{1 - \cos A}{\sin A} = \frac{\sin A}{1 + \cos A}$$

(a) Find $\sin 60^\circ$ by using a double-angle formula.

(b) If $\sin A = \frac{4}{5}$, $0^\circ < A < 90^\circ$, find $\sin 2A$, $\cos 2A$ and $\tan 2A$.

(c) Find $\cos 45^\circ$ from functions of 90°.

(d) If $\tan B = \frac{12}{13}$, $0^\circ < B < 90^\circ$, find $\tan 2B$.

Find $\tan(\frac{B}{2})$.

(e) If $\cos A = \frac{4}{5}$, $270^\circ < A < 360^\circ$, find $\cos(\frac{A}{2})$.

Find $\cos 2A$.

(f) If $\sin A = \frac{-5}{13}$, $270^\circ < A < 360^\circ$, find $\sin 2A$.

Find $\sin(\frac{A}{2})$.

V. D. CONTENT: Trigonometric Functions of Any Angle; Identities

OBJECTIVE: The student will be able to verify that expressions are identities by using trigonometric formulas.

ACTIVITIES: Verify each of the following identities:

(a) $\tan^2\theta \cos^2\theta + \cot^2\theta \sin^2\theta = 1$

(b) $\frac{\sin\theta}{1+\cos\theta} + \frac{\cos\theta}{\sin\theta} = \csc\theta$

(c) $\frac{\sec^2\theta - 6\tan\theta + 7}{\sec^2\theta - 5} = \frac{\tan\theta - 4}{\tan\theta + 2}$

(d) $\sec^4\theta - \sec^2\theta = \frac{1}{\cot^4\theta} + \frac{1}{\cot^2\theta}$

(e) $\frac{1}{2}\sin 2\theta = \frac{\tan\theta}{1+\tan^2\theta}$

(f) $\frac{\sin\theta + \sin 3\theta}{\cos\theta + \cos 3\theta} = \tan 2\theta$

(g) $1 + \cos 2\theta = \frac{2}{1+\tan^2\theta}$

(h) $\cot\theta = \frac{\sin 2\theta}{1-\cos 2\theta}$

(i) $\csc\theta \sec\theta = 2\csc 2\theta$

VI. A

CONTENT: Inverse Trigonometric Relations; General Values

OBJECTIVE: The student will be able to find general values of inverse relations.

ACTIVITIES:

(a) Find all angles θ for which:

(1) $\theta = \arcsin \frac{1}{2}$

(2) $\theta = \arccos \frac{\sqrt{2}}{2}$

(3) $\theta = \operatorname{arcsec} 2$

(4) $\theta = \arccos \left(-\frac{\sqrt{3}}{2}\right)$

(5) $\theta = \arctan (-1)$

(6) $\theta = \arctan .4142$

(b) Assume that all angles are <u>acute</u> and find:

(1) $\sin \left(\arcsin \frac{\sqrt{3}}{2}\right)$

(2) $\tan \left(\arctan \frac{2}{3}\right)$

(3) $\tan \left(\arcsin \frac{4}{5}\right)$

(4) $\sin (\arctan 1) + \cos \left(\arctan \frac{\sqrt{3}}{3}\right)$

VI. B.

CONTENT: Inverse Trigonometric Relations; Graphs of Inverse Trigonometric Functions

OBJECTIVE: The student will be able to graph inverse trigonometric functions.

ACTIVITIES:

(a) Sketch the graph of $y = \operatorname{Arcsin} x$. Identify the domain and range of the functions.

(b) Sketch the graph of $y = \operatorname{Arccos} x$. Identify the domain and range of the function.

(c) Sketch the graph of $y = \operatorname{Arctan} x$. Identify the domain and range of the function.

VI. C.

CONTENT: Inverse Trigonometric Relations; Principal Values of Inverse Functions

OBJECTIVE: The student will be able to find principal values of inverse trigonometric functions.

ACTIVITIES: Find the principal value of each of the following:

(a) Arcsin $\frac{1}{2}$

(b) Arccos $\frac{(-\sqrt{2})}{2}$

(c) Arctan (-1)

(d) Arcsin (-.5125)

(e) $\text{Cos}^{-1}\ (\frac{\sqrt{3}}{2})$

(f) $\text{Tan}^{-1}\ 4$

VII. A. CONTENT: Trigonometric Equations; Principal Values

OBJECTIVE: The student will be able to find principal values of trigonometric equations.

ACTIVITIES:

Find the principal values of each of the following:

(a) $2 \sin x - 1 = 0$

(b) $\sqrt{3} \tan x = -1$

(c) $\sin^2 x - 3 \sin x + 2 = 0$

(d) $2 \sin x \cos x - \cos x = 0$

(e) $\sec x \tan x - \tan x + 2 \sec x - 2 = 0$

VII. B. CONTENT: Trigonometric Equations ; General Values

OBJECTIVE: The student will be able to find general values of trigonometric equations.

ACTIVITIES:

Find the general values of the following equations:

(a) $y = \text{arcsec } 2$

(b) $3 \tan x - \cos x = 0$

(c) $\sin x + \cos x \sin x = 0$

(d) $2 \sin^2 x - \sin x + 1 = 0$

(e) $\sqrt{3} \sec x \sin x - 2 \sec x + \sqrt{3} \sin x = 2$

(f) $\cos x - \sin x = 1$

(g) $2 \sin^2 x - \cos x - 1 = 0$

(h) $\sin^2 x - 2 \sin x - 1 = 0$

VII. C. CONTENT: Trigonometric Equations; Restricted Solutions

OBJECTIVE: The student will be able to find restricted solutions of trigonometric equations.

ACTIVITIES: Solve for x, $0 \leq x < 360^\circ$

(a) $\sin (x - 30) = \frac{1}{2}$

(b) $5 \sec^2 x + 2 \tan x - 8 = 0$

(c) $2 \cos 2x - \sin x + 1 = 0$

(d) $2 \sin x \tan x + 2 \sin x - \tan x - 1 = 0$

(e) $\tan^3 x - 3 \tan x = 0$

(f) $\sin x - 2 \cos x = 1$

(g) $\sin 2x \cos x + \cos 2x \sin x = \frac{1}{2}$

(h) $\sin^2 x - \cos^2 x = 0$

VIII. A. CONTENT: Applications of Trigonometry; Right Triangles

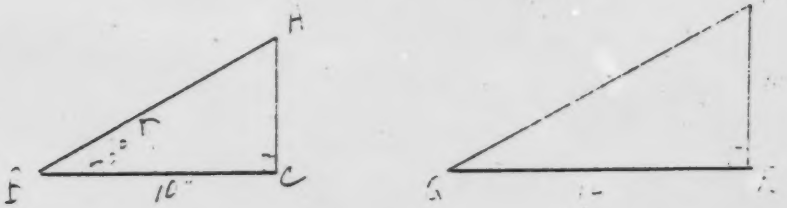
OBJECTIVE: The student will be able to solve right triangles given

(a) Two sides;

(b) An angle and a side.

ACTIVITIES:

(a) Solve each right triangle.



(b) The measure of each base angle of an isosceles triangle is 72 degrees. If the length of the base is 10 cm, find the length of the altitude to the base and the length of the congruent sides of the triangle.

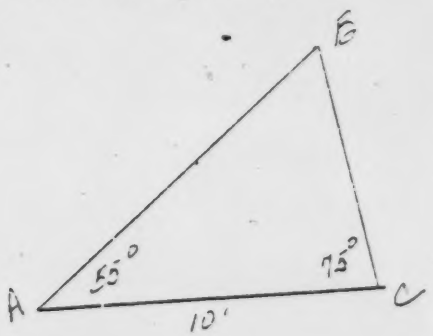
(c) A monument 200 feet high casts a shadow 325 feet long. Find the angle of elevation of the sun.

VIII. B. CONTENT: Applications of Trigonometry; Law of Sines and Law of Cosines

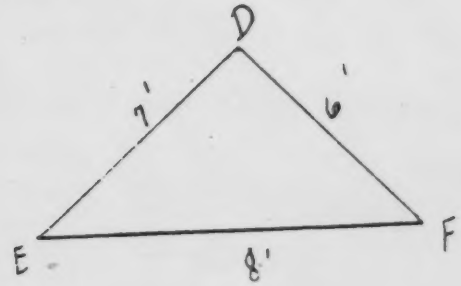
OBJECTIVE: The student will be able to solve triangles by applying the Law of Sines and the Law of Cosines.

ACTIVITIES:

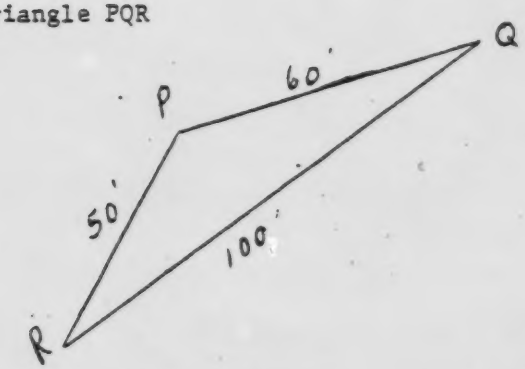
(a) Solve triangle ABC.



(b) Solve triangle DEF.



(c) Solve triangle PQR



VIII. C. CONTENT: Application of Trigonometry ; Use of Calculators

OBJECTIVE: The student will be able to perform calculations involving trigonometric functions using the calculator.

ACTIVITIES:

(a) Given triangle ABC:

(1) a = 22.34

(2) b = 13.74

(3) c = 10.15

Find angle B.

(b) Given triangle ABC:

(1) A = 70° 13'

(2) B = 52° 48'

(3) a = 640

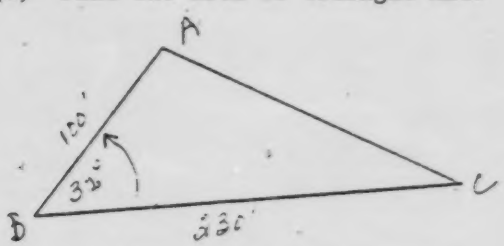
Find side b.

VIII. D. CONTENT: Applications of Trigonometry; Area of Triangles

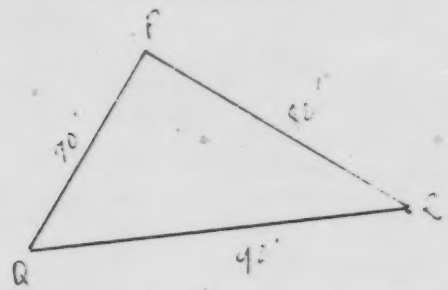
OBJECTIVE: The student will be able to find the area of a triangle given the lengths of three sides or of two sides and the measure of the included angle of the triangle.

ACTIVITIES:

(a) Find the area of triangle ABC.



(b) Find the area of triangle PQR.



IX. A. CONTENT: Polar Coordinates; Conversion

OBJECTIVE: The student will convert from rectangular to polar coordinates and vice-versa.

ACTIVITIES:

(a) Convert to polar coordinates

(1) (1,1)

(2) (3,-4)

(b) Convert to rectangular coordinates.

(1) $(2,\frac{5\pi}{6})$

(2) $(-3,210^{\circ})$

IX. B. CONTENT: Polar Coordinates; Graphing

OBJECTIVE: The student will sketch graphs of relations in polar form

ACTIVITIES:

(a) $r = 2 \cos \theta$

(b) $r = 2(1 + \cos \theta)$

(c) $r = 2 \sin 2 \theta$

X. A. CONTENT: Complex Numbers; Definition and Properties

OBJECTIVE: The student will be able to:

(a) Define complex, numbers;

(b) Perform fundamental operations on complex numbers.

ACTIVITIES:

(a) Write each of the following in the form of $x + yi$

(1) $(i)^6$

(2) $(1 + i)(2 - i)$

(3) $(2 + 3i) - (4 - 6i)$

(4) $\frac{1 - i}{5 - 2i}$

(5) $(2 - 3i)^2$

(6) $|3 - i|$

(b) Solve for x and y

(1) $3x + 15i = 6 - yi$

(2) $x - yi = 2 + 3i$

(3) $x + y - 4 = - xi + 2 yi + i$

(4) $x + yi = (3 + i)(1 - 3i)$

(5) $2x + y + (3x - y) i = 4 + i$

X. B. CONTENT: Complex Numbers; Rectangular and Polar Forms

OBJECTIVE: The student will be able to write complex numbers in rectangular and in polar form.

ACTIVITIES:

(a) Represent each complex number Z as a point in the plane and find the amplitude and modulus. Express the complex number in polar form.

(1) $Z = 1 + i$

(2) $Z = \sqrt{3} - 3i$

(3) $Z = 3i$

(4) $Z = 5$

(5) $Z = -1 + i$

(6) $Z = \frac{1}{2} + i\sqrt{\frac{3}{2}}$

(7) $Z = 1$

X. C. CONTENT: Complex Numbers; Multiplication and Division in Polar Form

OBJECTIVE: The student will be able to multiply and divide complex numbers in polar form.

ACTIVITIES:

(a) Perform the indicated operations and express each answer in rectangular form.

(1) $2(\cos 30^\circ - i \sin 30^\circ) \cdot 3(\cos 30^\circ + i \sin 30^\circ)$

(2) $\frac{8(\cos 135^\circ + i \sin 135^\circ)}{2(\cos 45^\circ + i \sin 45^\circ)}$

(3) $(5 \text{ cis } 80^\circ) \cdot (4 \text{ cis } 40^\circ)$

(4) $(6 \text{ cis } 85^\circ) \div (3 \text{ cis } 25^\circ)$

(5) $(10 \text{ cis } 90^\circ) \div (2 \text{ cis } 45^\circ)$

(b) Express each complex number in polar form and then perform the indicated operations.

(1) $(1 + i)(1 - i\sqrt{3})$

(2) $(2 - 2i)(1 + \sqrt{3}i)$

(3) $\frac{\sqrt{3} + 3i}{2 - 2i}$

(4) $\frac{(1 - i)(-1 + i\sqrt{3})}{(\sqrt{3} + i)}$

(5) $(1 + i)(-2 + 2i)$

(6) $\frac{1 + i}{1 - i}$

(7) $\frac{-1 + i}{1 + \sqrt{3}i}$

X. D.

CONTENT: Complex Numbers; DeMoivre's Theorem

OBJECTIVE: The student will be able to use DeMoivre's Theorem to raise complex numbers to integral powers.

ACTIVITIES: Use DeMoivre's Theorem and express the following complex numbers in rectangular form:

(a) $(2 \text{ cis } 20^\circ)^3$

(b) $(3 \text{ cis } 15^\circ)^2$

(c) $(\text{cis } 18^\circ)^5$

(d) $(\sqrt{3} + i)^4$

(e) $(-i)^7$

(f) $(1 + i)^4$

(g) $\left(\frac{-1 + i\sqrt{3}}{2}\right)^7$

(h) $(-\sqrt{3} + i)^6$

X. E.

CONTENT: Complex Numbers; Roots of Complex Numbers

OBJECTIVE: The student will be able to find all roots of complex numbers.

ACTIVITIES:

(a) Find the 3 cubic roots of 8i. Express each in rectangular form.

(b) Find the 4 fourth roots of 16. Express each in rectangular form.

(c) Find the 4 fourth roots of $-8 - 8\sqrt{3}i$. Express each in rectangular form.

(d) Find the 3 cubic roots of $-1 + i$.

(e) Find the fifth roots of -1.

XI. A. CONTENT: Relations and Functions; Relation

OBJECTIVE: The student will be able to:

(a) Define a relation;

(b) Identify domain and range

ACTIVITIES:

(a) State the definition of a relation

(b) State domain and range of the following relations over the set of reals:

(1) $\{(3,4), (2,5), (1,4), (-30)\}$

(2) $2x + 4 = y$

(3) $\sqrt{x-1} = y$

(4) $y = \frac{x + 3}{x + 1}$

(5) $\frac{x^2 - 1}{x + 1} > y$

XI. B. CONTENT: Relations and Functions; Linear Relations and Graphs

OBJECTIVE: The student will be able to define a linear relation and sketch its graph.

ACTIVITIES:

(a) Select the linear relations:

(1) $x^2 + 2 = y$

(2) $\sqrt{x} = y$

(3) $2x + y = 5$

(4) $\sqrt{2x} + 5 = y$

(5) $\frac{1}{x} = y - 1$

(b) Sketch the graph of the following:

(1) $5x + 2y = 6$

(2) $2x < 3y + 7$

(3) $3 < 2x + y = 7$

(4) $|2x - 2| < 3$

(5) $y > |x-1|$

(6) $|x| + |y| \leq 3$

$|x - y| < -3$

XI. C. 1,2 CONTENT: Relations and Functions; Functions

OBJECTIVE: The student will be able to:

(a) Differentiate a function from a group of relations given a graph, set of ordered pairs, an equation and a mappin;

(b) Sketch the graph and state the domain and range of special functions.

ACTIVITIES:

(a) Which of the following are functions of x:

(1) $\{(3,4), (5,6), (-3,-4), (-2,6)\}$

(2) $\{(4,5), (3,6), (4,-1)\}$

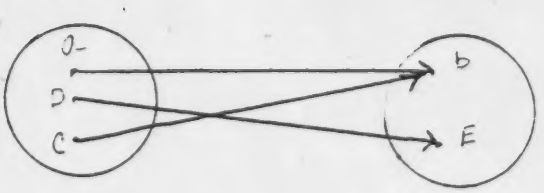
(3) $|x + 3| = y$

(4) $2|y| + 3 = 6$

(5)

(6)

(7)



(b) Sketch:

(1) $F(x) = \begin{cases} 3, & \text{if } x \geq 3 \\ x + 2, & \text{if } -1 < x < 3 \\ -2, & \text{if } x \leq -1 \end{cases}$

(2) $f(x) = [\![x]\!] + 1$

(3) $F(x) = \begin{cases} x^2, & \text{if } 0 \leq x \leq 2 \\ x, & \text{if } x > 2 \end{cases}$

XI. D. CONTENT: Relations and Functions; Arithmetic of Functions

OBJECTIVE: The student will be able to:

(a) Perform the fundamental arithmetic operations on any two functions;

(b) Graph the sum and difference of two functions.

ACTIVITIES:

(a) Given $f(x) = 2x + 1$ and $g(x) = 2x^2 + 3$, find:

(1) $f + g$

(2) $f - g$

(3) $f \times g$

$\frac{f}{g}$

(b) Given $f(x) = x^2$ and $g(x) = x - 2$, graph: $f + g$

XI. E. CONTENT: Relations and Functions; Composition and Inversion of Functions

OBJECTIVE: The student will be able to:

(a) Find the composition of functions;

(b) State the inverse of a given function.

ACTIVITIES:

(a) Given $f(x) = 2x - 7$ and $g(x) = x^2 + 1$ find

(1) $f(g(x))$

(2) $g(f(x))$

(b) Given: $g = \{(1,2), (3,-4), (0,3), (-1,2)\}$

$h = \{(1,3), (0,-1), (3,4), (-2,2)\}$

Find: (1) $g \circ h$

(2) $h \circ g$

(c) Given the following functions

1 State the inverse.

2 Is $f^{-1}(x)$ a function? If not restrict $f^{-1}(x)$ so that it is a function.

a. $f(x) = x^2$

b. $f(x) = 3x + 2$

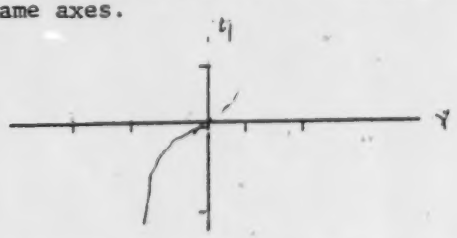
c. $f(x) = \sqrt{x - 4}$

(d) Are the following inverses of each other?

(1) $y = \frac{1}{3}(x + 2)$; $y = 3x - 2$.

(2) $y = x - 2$; $y = 2 - x$.

(e) Sketch the inverse of the graph below on the same axes.



XI. F. CONTENT: Relations and Functions ; Exponential and Logarithmic Functions

OBJECTIVE: The student will be able to:

(a) Change from exponential to logarithmic form and conversely;

(b) Sketch exponential and logarithmic functions.

ACTIVITIES:

(a) Express the following using logarithmic notation:

(1) $2^3 = 8$

(2) $16^{\frac{1}{4}} = 4$

(3) $e^x = q$

(4) $4^3 = 64$

(b) Express the following using exponential notation.

(1) $\log_2 32 = 5$

(2) $\log_4 x^2 = 4$

(3) $\log_3 8 = x$

(c) Sketch $y = 2^x$ and its inverse.

(d) Express y in terms of x if: $\log y = 2 \log x$.

XII. A, B.

CONTENT: Polynomial Functions; Definitions; Factor and Remainder Theorems

OBJECTIVE: The student will be able to:

(a) Define a polynomial in x;

(b) Apply the Factor and Remainder Theorems.

ACTIVITIES:

(a) Use the Remainder Theorem to find the remainder if the first expression is divided by the second expression.

(1) $x^3 + 7x^2 - 3x - 2$; $x - 1$

(2) $-3x^3 + 5x^2 - 2x + 1$; $x + 2$

(3) $x^{27} - 5$; $x + 1$

(b) Use the Factor Theorem to determine which of the following are true.

(1) $x - 2$ is a factor of $x^3 - x^2 - 5x + 6$

(2) $x + 2$ is a factor of $x^4 - 16$.

(3) $x - y$ is a factor of $x^n - y^n$ if n is any natural number.

(c) Find the value of k for which $x - 3$ is a factor of $kx^3 - 6x^2 + 2kx - 12$.

XII. C.

CONTENT: Polynomial Functions; Synthetic Division

OBJECTIVE: The student will be able to use synthetic division to find quotients and remainders.

ACTIVITIES:

(a) Use synthetic division to find the quotient and remainder.

(1) $(x^3 - 2x^2 + 3x - 5) \div (x - 3)$

(2) $(3x^3 - x + 1) \div (x - 5)$

(3) $(x^4 + 2x^3 - 5x^2 - 4x + 6) \div (x + \sqrt{2})$

(b) (1) If $f(x) = x^3 - 2x^2 + 3x + 4$, find $f(3)$ and $f(-2)$

(2) If $f(x) = 2x^4 - 4x^3 + x^2 - 6x - 3$, find $f(1 + \sqrt{2})$

(c) Use synthetic division to find a value of k so that -2 is a root of $3x^3 + 5x^2 + kx - 10 = 0$.

XII. D. CONTENT: Polynomial Functions; Rational Root Theorem

OBJECTIVE: The student will be able to:

(a) Use the Rational Root Theorem to find roots of equations.

ACTIVITIES:

(a) Solve each equation given that:

(1) 3 is a root of $x^3 - 4x^2 + x + 6 = 0$.

(2) 1 and $-\frac{2}{3}$ are roots of $3x^4 + 14x^3 - 4x^2 - 11x - 2 = 0$.

(3) $-\frac{1}{3}$ is a double root of $9x^4 + 24x^3 + 49x^2 + 26x + 4 = 0$.

(b) Find all roots of:

(1) $9x^4 - 3x^3 + 7x^2 - 3x - 2 = 0$.

(2) $2x^4 + 5x^3 + 6x^2 + 2x = 0$.

XII. E., F. CONTENT: Polynomial Functions; Location Theorem; Graphing

OBJECTIVE: The student will be able to:

(a) Locate real roots of equations between two consecutive integers;

(b) Graph higher degree equations.

ACTIVITIES:

(a) Find two consecutive integers between which a negative root of $2x^3 + 5x^2 + 6x + 15 = 0$ lies.

(b) Find two consecutive integers between which a positive root of $2x^3 - 3x^2 + 8x - 12 = 0$ lies.

(c) Use the Location Theorem to sketch the graph of $f(x) = -8x^3 + 14x^2 - 23x - 6$.

XII. G., H. CONTENT: Polynomial Functions; Descartes Rules of Signs; Upper and Lower Bound Theorems

OBJECTIVE: The student will be able to:

(a) Apply Descartes Rules of Signs to determine the nature of roots of equations;

(b) Use the Upper and Lower Bound Theorems to aid in solving equations.

ACTIVITIES:

(a) Find the maximum number of positive and negative real roots of:

(1) $4x^3 + 3x^2 - x + 1 = 0$.

(2) $x^3 + x^2 + 6 = 0$.

(3) $2x^4 + x^3 - 5x^2 - 3x + 5 = 0$.

(b) Supply the missing information.

(1) $x^6 + 2x^4 + 2 = 0$ has __?__ non-real complex roots.

(2) $2x^5 + x^3 + x + 1 = 0$ has __?__ non-real complex roots.

(3) $2x^3 - x^2 - 4 = 0$ has __?__ non-real complex roots.

(c) Find an upper and lower bound for the roots of $f(x) = x^4 + x^3 + 70x^2 - 2x - 144 = 0$.

XII. I. CONTENT: Polynomial Functions; Writing Equations

OBJECTIVE: The student will be able to write equations of minimum degree that have selected solutions.

ACTIVITIES:

(a) Write an equation (of minimum degree) that has real coefficients and $1 + 2i$ and 4 as roots.

(b) Write an equation with integral coefficients that has $(1 + i)$ as a root and $-\frac{2}{3}$ as a double root.

(c) Write an equation (of minimum degree) that has rational coefficients and $2 + \sqrt{3}$ and 4 as roots.

XIII. A., B., C., and D.

CONTENT: Conic Sections; Circles; Parabolas; Ellipses; Hyperbolas

OBJECTIVE: The student will be able to:

(a) Identify, write and sketch conic sections;

(b) Identify any conic section of the form $Ax^2 + Bxy + Cy^2 + dx + Ey + F = 0$.

ACTIVITIES:

Given the following:

(a) Identify the conic;

(b) Write equation in standard form;

(c) Sketch.

(1) $4x^2 + 4y^2 - 16 = 0$.

(2) $x^2 - 2x - y = 0$.

(3) $4x^2 - 9y^2 - 36 = 0$.

(4) $9x^2 + 4y^2 - 36 = 0$.

(5) $x^2 + y^2 - 2x - 8 = 0$.

(6) $9x^2 + 4y^2 - 18x + 16y - 11 = 0$.

(7) $xy - 1 = 0$.

XIV. A., B.

CONTENT: Sequences and Series; Definition and Notation

OBJECTIVE: The student will be able to:

(a) Define a sequence and a series;

(b) Use general term form to write sequences and series.

ACTIVITIES:

(a) Complete each sequence through seven terms and then write the sequence in general term form.

(1) $-1, -4, -7, -10, ___, ___, ___.$

(2) $\sqrt{2}, 2, 2\sqrt{2}, 4, ___, ___, ___.$

(3) $\frac{1}{2}, \frac{2}{4}, \frac{3}{8}, \frac{4}{16}, ___, ___, ___.$

(b) Write each series in shorthand notation.

(1) $-1 + 1 - 1 + 1 - 1 + \ldots$

(2) $2 - 4 + 8 - 16 + \ldots - 256.$

(3) $1 + \frac{2}{3} + \frac{3}{5} + \frac{4}{7} + \ldots \frac{9}{17}.$

(4) $-1 + 1 - 3 + 5 - 7 + \ldots 17.$

(c)

(1) $\sum_{K=I}^{8} 2K = _____$

(2) $\sum_{K=1}^{5} 3K - 1 = _____$

XIV. C.

CONTENT: Sequences and Series ; Recursion Formulas

OBJECTIVE: The student will be able to write sequences by using recursion formulas.

ACTIVITIES:

(a) Write the first five terms of each sequence if:

(1) $a_1 = 3$ and $a_{n+1} = 2a_n + 4$

(2) $a_1 = 2$ and $a_{n+1} = \left(\frac{2n}{n+1}\right) a_n$

(b) Find a recursion formula for each sequence by identifying a_1 and writing a_{n+1} in terms of a_n.

(1) 3, 6, 9, 12, 15, 18 . . .

(2) 2, 6, 10, 14 . . .

(3) 1, 3, 7, 15, . . .

XIV. D. CONTENT: Sequences and Series; Arithmetic Sequences and Series

OBJECTIVE: The student will be able to find missing terms in arithmetic sequences and series.

ACTIVITIES:

(a) Find the sum of all even integers between 5 and 75.

(b) Which term of 8, 5, 2, . . . is -28?

(c) If the fifth term of an arithmetic sequence is -16 and the twentieth term is -46, find the twelfth term.

(d) The sum of the first three terms of an arithmetic series is 8. If the first term is 2, find the common difference and the seventh term.

(e) Find three arithmetic means between 11 and 23.

XIV. E. CONTENT: Sequences and Series; Geometric Sequences and Series

OBJECTIVE: The student will be able to find missing terms in geometric sequences and series.

ACTIVITIES:

(a) Which term of $\frac{3}{2}$, -3, 6, . . . is 96?

(b) The sum of eight terms of a geometric series is 640. If the common ratio is 2, find the first term.

(c) Find y so that y + 5, y - 1, y - 3 is geometric.

(d) The first term of a geometric series is 4 and the last term is 324. If the common ratio is 3, find the sum of these terms.

(e) The fifth term of a geometric sequence is 1. If the common ratio is $\frac{1}{2}$, find the first term and the sum of the first five terms.

XIV. F.

CONTENT: Sequences and Series; Infinite Geometric Series

OBJECTIVE: The student will be able to find the sum of infinite geometric series whose common ratio (r) has the property that $|r| < 1$.

ACTIVITIES:

(a) Find the sum of the following infinite geometric series.

(1) $10 + 5 + \frac{5}{2} + \ldots$

(2) $\frac{3}{4} - \frac{1}{2} + \frac{1}{3} - \frac{2}{9} + \ldots$

(3) $3 - 0.3 + 0.03 - 0.003 + \ldots$

(b) Write each decimal expression in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

(1) $.2\overline{323}$

(2) $.288\overline{8}$

(3) $.134\overline{242}$

(c) Find the sum of each series.

(1) $\sum_{n=1}^{\infty} (-\frac{1}{8})^n$

(2) $\sum_{n=1}^{\infty} 2(\frac{1}{4})^{n-1}$

(3) $(\frac{1}{2} + \frac{1}{3}) + (+\frac{1}{4} + \frac{1}{9}) + \ldots + (\frac{1}{2n} + \frac{1}{3n}) + \ldots$

(d) Suppose a ball rebounds $\frac{3}{5}$ of the distance it falls.

If it is dropped from a height of 20 ft., how far does it travel before coming to rest?

(e) Find two geometric means between 16 and 54.

XIV. G.

CONTENT: Sequences and Series; Limit of a Sequence

OBJECTIVE: The student will be able to find the limit of an infinite sequence.

ACTIVITIES:

For each part, determine if the sequence converges or diverges by finding $\lim_{n \to \infty} a_n$

(a) $1, -1, 1, -1, \ldots (-1)^{n+1}, \ldots$

(b) $\left\{\frac{n}{2n-7}\right\}_{n=1}^{\infty}$

(c) $\left\{\frac{3n^2 + 5n - 2}{2n^2}\right\}_{n=1}^{\infty}$

(d) $\left\{3 - \frac{1}{10^n}\right\}_{n=1}^{\infty}$

(e) $\left\{\frac{3n}{n+1} - \frac{n+3}{3n}\right\}_{n=1}^{\infty}$

(f) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots$

XIV. H. CONTENT: Sequences and Series; Convergent and Divergent Series

OBJECTIVE: The student will be able to determine if selected infinite series converge or diverge.

ACTIVITIES:

(a) Prove that each series diverges by showing that $\lim_{n\to\infty} a_n \neq 0$

(1) $\sum_{n=1}^{\infty} 2n + 3$

(2) $\sum_{n=1}^{\infty} \frac{n^3 + 2}{n^3 + 1}$

(3) $\sum_{n=1}^{\infty} \frac{2n^2 - 3n + 4}{5n^2 + n + 2}$

(b) Use the ratio test to determine which series converge.

(1) $\sum_{n=1}^{\infty} \frac{1}{n!}$

(2) $\sum_{n=1}^{\infty} \frac{n}{2^n}$

(c) Use any test you want to determine if

$\sum_{n=1}^{\infty} \frac{1}{4n^2-1}$ converges.

If the series converges, what does it converge to?

XIV. I.

CONTENT: Sequences and Series ; Mathematical Induction

OBJECTIVE: The student will be able to use the principle of mathematical induction to solve selected exercises.

ACTIVITIES:

Use the principle of mathematical induction to prove each of the following.

(a) $1 + 4 + 7 + \ldots + (3n - 2) + \ldots = \frac{n(3n-1)}{2}$.

(b) $3 + 3^2 + 3^3 + \ldots + 3^n + \ldots = \frac{3^{n+1} - 3}{2}$.

(c) $1 + 2 + 3 + \ldots + n + \ldots < \frac{1}{8}(2n + 1)^2$.

(d) $2^{n+3} < (n + 3)!$ for any positive inter n.

(e) $6\ (7^n - 1)$ for any positive inter n.

XIV. J.

CONTENT: Sequences and Series ; Binomial Theorem

OBJECTIVE: The student will be able to use the Binomial Theorem to expand binomials.

ACTIVITIES:

(a) Expand $(2x - y)^5$.

(b) Find the fourth term of $(x - 3y)^{11}$.

(c) Find the seventh term of $(x + \frac{2}{x})^9$.

(d) Find the term involving x^4 in the expansion of $(3x^2 - \frac{2}{x})^8$.

XV. A., B. CONTENT: Matrices; Definition; Sums and Products of Matrices

OBJECTIVE: The student will be able to:

(a) Define matrix;

(b) Find sums and products of matrix arrays.

ACTIVITIES:

(a) If $a = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, find:

(1) A + B

(2) A - B

(3) A · B

(b) If $A = \begin{bmatrix} 3 & 2 & -1 \\ 4 & -3 & 1 \\ -3 & 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ find A·B and A^2 if they exist.

(c) Find $2 \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - 1 \begin{bmatrix} 0 & -2 \\ 3 & 4 \end{bmatrix}$

XV. C. CONTENT: Matrices; Transpose of a Matrix

OBJECTIVE: The student will be able to find the transpose of a matrix.

ACTIVITIES:

(a) If $A = \begin{bmatrix} 3 & -5 \\ 4 & 1 \end{bmatrix}$, find A^T

(b) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$ prove that

$(A + B)^T = A^T + B^T$.

XV. D. CONTENT: Matrices; Determinant of a Matrix

OBJECTIVE: The student will be able to find the determinant of a matrix by using reduction by minors and cofactors.

ACTIVITIES:

(a) Find $\sigma(A)$

(1) $A = \begin{bmatrix} 3 & -5 \\ 2 & 3 \end{bmatrix}$

(2) $A = \begin{bmatrix} 3 & -3 & -4 \\ 5 & -2 & -1 \\ -2 & 3 & -1 \end{bmatrix}$

(b) If A and B are 2 x 2 matrices, prove that

$\sigma(A^T) = \sigma(A)$

XV. E. CONTENT: Matrices; Inverse of Matrices

OBJECTIVE: The student will be able to use the inverse of a matrix to solve linear matrices.

ACTIVITIES:

(a) Find the inverse of each matrix if it exists.

(1) $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

(2) $A = \begin{bmatrix} 2 & -3 \\ 4 & 3 \end{bmatrix}$

(3) $A = \begin{bmatrix} -2 & -3 \\ 4 & 6 \end{bmatrix}$

(b)

(1) If $\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix} \cdot A = \begin{bmatrix} 4 & -6 \\ 15 & 22 \end{bmatrix}$, find A.

(2) If $\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix} \cdot A = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix}$, find A.

(3) If $A = \begin{bmatrix} 3 & 6 \\ -2 & 2 \end{bmatrix}$, find A^{-1}.

(4) If $\begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -8 \\ -3 \end{bmatrix}$ find x and y.

XV. F. CONTENT: Matrices; Systems of n Equations and n Variables

OBJECTIVE: The student will be able to solve linear systems of n equations in n variables.

ACTIVITIES:

Solve each system of equations.

(a) $x + y + z = 6$

$x + y = 3$

$3z + w = 13$

$2x + y + w = 8$

(b) $x + y + z = 3$

$2x - y - z = 3$

$3x + 2y + 5z = 20$

*XVI. A

CONTENT: Probability; Permutations

OBJECTIVE: The student will be able to:

(a) Define a permutation;

(b) Solve problems associated with permutations.

ACTIVITIES:

(a) Evaluate

(1) ${}_6P_6$

(2) ${}_8P_4$

(b) How many three digit-numbers can be written from digits 2, 3, 4, 7, and 9 if:

(1) repetition of digits is permitted?

(2) repetition of digits is not permitted?

(c) A president, a vice president, a secretary and a treasurer are to be selected from a class of thirty. In how many ways can they be selected if no member can hold two offices?

(d) How may arrangements of the letters of the word "LOUISIANA" are possible?

*XVI. B.

CONTENT: Probability; Combinations

OBJECTIVE: The student will be able to solve problems associated with combinations.

ACTIVITIES:

(a) Evaluate.

(1) ${}_7C_2$

(2) ${}_{15}C_7$

(b) In how many ways can a committee of 4 be selected from a class of 12?

(c) How many subsets are there of a set that has 6 elements?

(d) A bag contains 6 red balls and 4 black balls. In how many ways can 2 red balls and 2 black balls be drawn?

*Optional

*XVI. C., D. CONTENT: Probability; Sample Space; Binomial Theorem

OBJECTIVE: The student will be able to:

(a) Define a sample space and complementary events;

(b) Use the Binomial Theorem to solve probability problems.

ACTIVITIES:

(a) If three half-dollars are tossed, find a sample space and identify the ordered triples.

(b) If a pair of dice is thrown, what is the probability that the sum of the numbers that are shown is seven?

(c) If a coin is tossed eight times, what is the probability of getting three heads and five tails? at least three heads?

*XVI E. CONTENT: Probability; Dependent and Independent Events

OBJECTIVE: The student will be able to solve probability problems that involve independent and dependent events.

ACTIVITIES:

(a) If a die and a coin are tossed, what is the probability that the die will show a five and the coin will show heads?

(b) A bag contains 4 red balls and 3 white balls. If two balls are drawn, one at a time without replacement, what is the probability that both balls will be red?

(c) If a pair of dice is thrown, what is the probability of getting at least one four?

*Optional

*XVII. A., B.

CONTENT: Vectors ; Definition

OBJECTIVE: The student will be able to:

(a) Define Cartesian products and construct Cartesian lattices for finite sets of integers;

(b) Specify a set given a rule or a roster.

ACTIVITIES:

(a) If $A = \{2, -1, 3\}$ and $B = \{-3, 4\}$, find AXB.

(b) Identify K by roster, given that

$K = \{(x,y) \mid (x + y, 2x - y) = (6,3)\}$

*XVII. C.

CONTENT: Vectors ; Displacements

OBJECTIVE: The student will be able to depict real numbers as displacements along a real number line.

ACTIVITIES:

Graphically represent the displacement from -6 to -2.

*XVII. D. (1, 2, 3, 4)

CONTENT: Vectors ; Vectors in a Plane

OBJECTIVE: The student will be able to:

(a) Recognize vectors in a plane as ordered pairs of real numbers, and as directed line segments;

(b) Add vectors;

(c) Construct vector diagrams;

(d) Diagram vector sums.

ACTIVITIES:

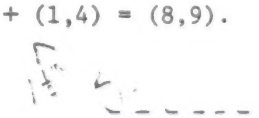
(a) Let $\overrightarrow{OT}$ represent (5,1) and $\overrightarrow{TS}$ represent (-2,1).

Name the ordered pair represented by $\overrightarrow{OS}$ and sketch each vector.

(b) If $\overrightarrow{RS}$ represents (3,-2) find S if R(1,-2).

(c) Find r and s so that (r,s) + (1,4) = (8,9).

(d) If V = $\overrightarrow{1 \text{ in}}$ and W =



construct the resultant V + W.

* Optional

*XVII. E.

CONTENT: Vectors; Norm of a Vector

OBJECTIVE: The student will be able to find the norm of a vector.

ACTIVITIES:

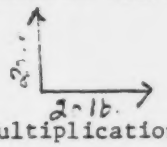
(a) Find the norm of each vector

(1) (0,-6)

(2) (-3,4)

(3) $(2\sqrt{3}, \sqrt{3})$

(b) Use the diagram to find the magnitude and direction of the resultant.



*XVII F.

CONTENT: Vectors; Multiplication of a Vector by a Scalar

OBJECTIVE: The student will be able to multiply a vector by a scalar and illustrate the result by a vector diagram.

ACTIVITIES:

(a) If v = (-6, 4) find the scalar multiple 2v.

Find the scalar multiple $-\frac{1}{2}$ v. Illustrate with diagrams.

(b) If v = (-4,2), find the vector norm of 2v.

(c) Which of the following illustrate vectors that are parallel?

(1) (6,10) and (3,5)

(2) (-8,4) and (16,-8)

(3) (8,-4) and (-4,-2)

(d)

(1) Find two vectors that are in the same direction.

(2) Find two vectors that are in opposite directions.

* Optional

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*XVIII. A., B

CONTENT: Differential Calculus; Limit of a Function; Continuous Function

OBJECTIVE: The student will be able to:

(a) Find limits of functions;

(b) Determine if a function is continuous at a point.

ACTIVITIES:

(a) Graph each function and by visual inspection find $\lim_{x \to 0} f(x)$, $\lim_{x \to 1} f(x)$ and $\lim_{x \to -1} f(x)$

(1)
$$f(x) = \begin{cases} 1 \text{ for } x = 0 \\ x^2 \text{ for } 0 < x < 1 \text{ or } -1 \leq x \leq 0 \\ 1 \text{ for } x > 1 \text{ or } x < -1 \end{cases}$$

(2)
$$f(x) = \begin{cases} x \text{ for } 0 \leq x < 1 \\ x^2 + 1 \text{ for } x \geq 1 \\ -x \text{ for } -1 \leq x < 0 \\ x + 1 \text{ for } x < -1 \end{cases}$$

(b) Find the domain and range of each function described in part (a).

(c) Determine if the functions described in part (a) are continuous at $x = -1$, $x = 0$ and $x = 1$.

(d) Define a function whose domain is {reals} and is discontinuous at infinitely many points.

*XVIII. C.

CONTENT: Differential Calculus; Limit Theorems

OBJECTIVE: The student will be able to use the limit theorems to find limits of functions.

ACTIVITIES:

Find the limit, if it exists.

(a) $\lim_{x \to 3} x^2 - 2x$

(b) $\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$

* Optional

(c) $\lim_{x \to 0} \frac{1}{x}$

(d) $\lim_{x \to \infty} \frac{1}{x}$

(e) $\lim_{x \to 2} \frac{2x^2 - 7x + 6}{x^2 + 3x - 10}$

(f) $\lim_{x \to \infty} \frac{3x^2 - 5x}{2x^2 + 6x - 5}$

(g) $\lim_{x \to \infty} (2 + \frac{3x^2}{2 + x^2})$

(h) $\lim_{x \to 3} \frac{x^3 - 3x^2 + x - 3}{x - 3}$

(i) $\lim_{x \to -2} \frac{2 - \sqrt{2 - x}}{x + 2}$

*XVIII. D. (1) CONTENT: Differential Calculus; Derivatives of Functions; Definition of Derivative

OBJECTIVE: The student will be able to find the derivative of a function by using the definition of the derivative of a function.

ACTIVITIES:

Use $f'(x) = \lim_{h \to o} \frac{f(x + h) - f(x) \text{ to}}{h}$

Solve each of the following.

(a) If $f(x) = 2x^2 - 3x$ find $f'(x)$

(b) If $f(x) = \frac{2}{x-1}$ find $f'(x)$ and $f'(3)$.

(c) If $f(x) = \frac{2x - 1}{x - 1}$ find $f'(x)$

(d) If $f(x) = 2x^3$ find an equation of the line that is tangent to $f(x)$ at $x - 2$.

(e) If $f(x) = mx + b$; m and b constants, find $f''(x)$.

* Optional

*XVIII. D. (2) CONTENT: Differential Calculus; Derivatives of Functions; Formulas

OBJECTIVE: The student will be able to find the derivative of functions by applying formulas.

ACTIVITIES:

Find f' (x) if:

(a) $f(x) = 2x^2 - 3x + 7$

(b) $f(x) = \frac{1}{x} - \frac{1}{x^2}$

(c) $f(x) = \sqrt[3]{x^2 + 6x + 3}$

(d) $f(x) = \sqrt{2x} + 2\sqrt{x}$

(e) $f(x) = x^2\sqrt{x}$

(f) $f(x) = \sqrt{2x - 1} \cdot \sqrt[3]{3x - 3}$

(g) $f(x) = \frac{\sqrt[3]{3x}}{2x}$

*XVIII. E., F. CONTENT: Differential Calculus; Local Maxima and Minima; Points of Inflection

OBJECTIVE: The student will be able to use derivaties of functions to find local maxima, local minima, and points of inflection.

ACTIVITIES:

For each of the following, find the critical points and identify local maxima, local minima and points of inflection for which f'(x) = 0.

(a) $f(x) = x^2 - 2x + 3$

(b) $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 6x + 8$

(c) $f(x) = x^4 - 2x^2 - 5$

(d) $f(x) = x^3 - 1$

*Optional

*XVIII. G. CONTENT: Differential Calculus; Graphing

OBJECTIVE: The student will be able to sketch functions by identifying local maxima and/or local minima of functions.

ACTIVITIES:

(a) Sketch the graphs of the functions in the previous section and find the domain and range of each.

(b) Sketch each of the following functions and identify all local maxima and minima. Also, find the domain and range of each function.

(1) $f(x) = \frac{x - 1}{x + 2}$

(2) $f(x) = \frac{-2x - 6}{x^2 - 4}$

*XVIII. H. CONTENT: Differential Calculus; Applications

OBJECTIVE: The student will be able to solve verbal problems by using the derivatives of functions.

ACTIVITIES:

(a) A projectile is thrown up from the ground with an initial velocity of 96 feet per second. Its height (in feet) after t seconds is $h = 96t - 16t^2$. Find the maximum height that is reached by the projectile.

(b) A rectangle has two of its vertices on the x axis and the other two above the x-axis and on the graph of $f(x) = 16 - x^2$. Find the dimensions of the rectangle of maximum area.

(c) A closed box is to have a square base. If the volume of the box is 64 cubic feet find the dimensions of the box so that the total area of the box is a minimum.

(d) Find the dimensions of the right circular cylinder of greatest volume that can be inscribed in a sphere whose radius is 9.

* Optional

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